On the evaluation of 3D wave-current models

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Friday 11th June 2010

Outline

Configuration

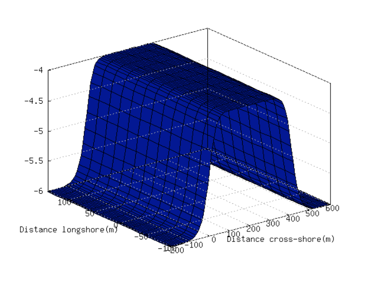


Figure: Bathymetry

- Test case from Ardhuin et al, 2008.
- **Simulation of the wave induced current for a case without wave breaking.**

Goals

- To test if the Mellor's model (JPO '03) is correctly implemented in coupled model.
- To test the Mellor's models (JPO '03 and JPO '08) ability to simulate the 3D oceanic circulation in presence of waves.
- To test the Ardhuin et al model (OM '08) ability to simulate the 3D oceanic circulation in presence of waves.

Formulation of Mellor's model (JPO '03)

The full set of equations:

$$
\begin{aligned}
\frac{\partial DU_\alpha}{\partial t} + \frac{\partial (DU_\alpha U_\beta)}{\partial x_\beta} + \frac{\partial (\Omega U_\alpha)}{\partial \zeta} + \epsilon_{\alpha\beta z} f_z DU_\beta \quad = \quad & -D\frac{\partial}{\partial x_\alpha}(g\widehat{\eta} + \widehat{p}_{atm}) \\
& -D^2 \int^0 \left(\frac{\partial b}{\partial x_\alpha} - \zeta \frac{\partial D}{\partial x_\alpha}\frac{\partial b}{\partial \zeta} \right) d\zeta \\
& -\frac{\partial S^{M03}_{\alpha\beta}}{\partial x_\beta} + \frac{\partial \overline{\tilde{s}_\alpha \tilde{p}}}{\partial \zeta} \\
& + \overline{\tilde{p}_{w\eta}\frac{\partial \tilde{\eta}}{\partial x_\alpha}} \frac{\partial F_{SS} F_{CC}}{\partial \zeta} - \frac{\partial}{\partial \zeta}\overline{\langle w' u'_\alpha \rangle}
\end{aligned}
$$

Formulation of Mellor's model (JPO '03)

Radiation stress tensor (horizontal part):

$$\frac{\partial S^{M03}_{\alpha\beta}}{\partial x_\beta} = \frac{\partial}{\partial x_\beta}\left[kDE\left(\frac{k_\alpha k_\beta}{k^2}F_{CS}F_{CC} - \delta_{\alpha\beta}(F_{CS}F_{CC} - F_{SS}F_{CS})\right)\right] + o(\epsilon^1)$$

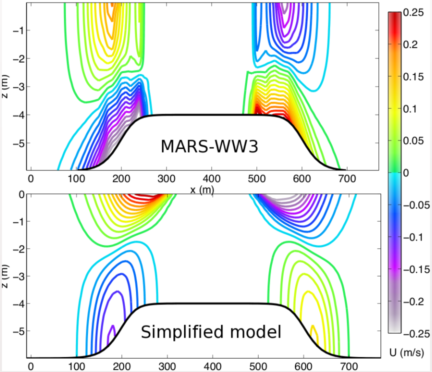
Radiation stress tensor (vertical part):

$$\frac{\partial}{\partial\zeta}\overline{\tilde{s}_\alpha\tilde{p}} \simeq \frac{\partial}{\partial\zeta}\left[(F_{CC} - F_{SS})E^{1/2}\frac{\partial}{\partial x_\alpha}(E^{1/2}F_{SS})\right] + O(\epsilon^1)$$

with:

$$F_{SS} = \frac{sinh(k\zeta D)}{sinh(kD)}, F_{CS} = \frac{cosh(k\zeta D)}{sinh(kD)}, F_{SC} = \frac{sinh(k\zeta D)}{cosh(kD)}, F_{CC} = \frac{cosh(k\zeta D)}{cosh(kD)}$$

Numerical results - Mellor JPO '03



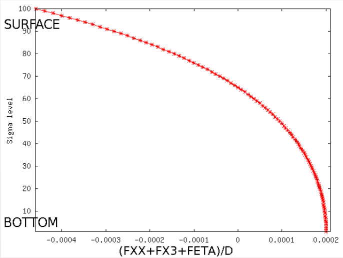
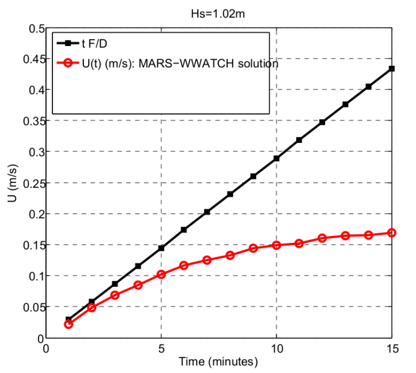


Figure: Zonal velocity from the coupled model (top panel) and from the temporal integration of $\frac{F}{D} - U\frac{\partial U}{\partial x}$ (bottom panel), $H_s = 1.02m$.

Numerical results - Mellor JPO '03



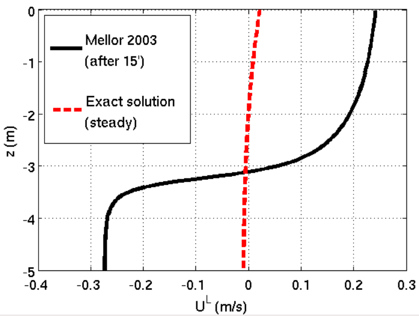


Figure: Zonal velocity from coupled model at the surface and in $x = 610m$ (red, green, blue lines). Linear tendance given by temporal integration of constant acceleration $\frac{F}{D}$ (black line). $H_s = 1.02\ m$.

Formulation of Mellor's model (JPO '08)

$$\frac{\partial DU_\alpha}{\partial t} + \frac{\partial (DU_\alpha U_\beta)}{\partial x_\beta} + \frac{\partial (\Omega U_\alpha)}{\partial \zeta} + \epsilon_{\alpha\beta z} f_z U_\beta = \ldots - D\frac{\partial S^{M08}_{\alpha\beta}}{\partial x_\beta} + \zeta \frac{\partial D}{\partial x_\beta}\frac{\partial S^{M08}_{\alpha\beta}}{\partial \zeta} \qquad (1)$$

$$\frac{\partial DU_\alpha}{\partial t} + \frac{\partial (DU_\alpha U_\beta)}{\partial x_\beta} + \frac{\partial (\Omega U_\alpha)}{\partial \zeta} + \epsilon_{\alpha\beta z} f_z DU_\beta = \ldots - \frac{\partial S^{M03}_{\alpha\beta}}{\partial x_\beta} + \frac{\partial \overline{\tilde{s}_\alpha \tilde{p}}}{\partial \zeta} \qquad (2)$$

Radiation stress tensor:

$$S_{\alpha\beta} = kE\left(\frac{k_\alpha k_\beta}{k^2} F_{CS} F_{CC} - \delta_{\alpha\beta} F_{SC} F_{SS}\right) + \delta_{\alpha\beta} E_D$$

with:

$$F_{SS} = \frac{sinh(k\zeta D)}{sinh(kD)},\ F_{CS} = \frac{cosh(k\zeta D)}{sinh(kD)},\ F_{SC} = \frac{sinh(k\zeta D)}{cosh(kD)}$$

and :

$$F_{CC} = \frac{cosh(k\zeta D)}{cosh(kD)},\ \ E_D = 0 \ \text{ if } \ z \neq \widehat{\eta} \ \text{ and } \int_{-h}^{\widehat{\eta}^+} E_D dz = \frac{E}{2}$$

Numerical results - JPO '08

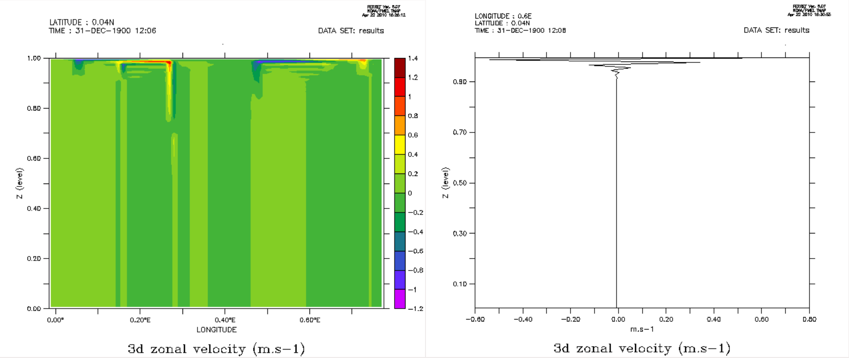


Figure: Map of zonal velocity (left panel) and vertical profile of zonal velocity (right panel) from coupled model with Mellor's Model (Mellor 2008).

Numerical results - JPO '08

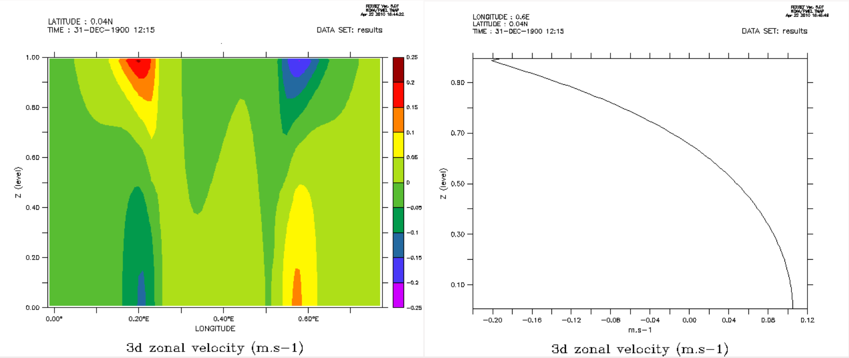


Figure: Map of zonal velocity (left panel) and vertical profile of zonal velocity (right panel) from coupled model with Mellor's Model (Mellor 2008).

Numerical results - JPO ’08

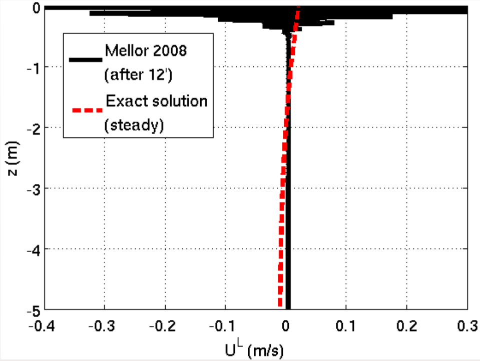


Figure: Vertical profiles

Conclusions

- The model of Mellor (JPO '03) is correctly implemented.
- The Mellor's models (JPO '03 and JPO '08) do not produce good results for the 3D oceanic circulation in presence of waves.

Equations

- **To solve the quasi-eulerian velocity with the coupled model** (Ardhuin et al 2008).

  Quasi-eulerian velocity: $(\widehat{u}, \widehat{v}, \widehat{w}) = (U, V, W) - (U_s, V_s, W_s)$

  The full set of equations:

$$
\begin{aligned}
\frac{\partial \widehat{u}}{\partial t} &+ \widehat{u}\frac{\partial \widehat{u}}{\partial x} + \widehat{v}\frac{\partial \widehat{u}}{\partial y} + \widehat{w}\frac{\partial \widehat{u}}{\partial z} - f\widehat{v} + \frac{1}{\rho}\frac{\partial p^H}{\partial x} - \widehat{F}_{m,x} \\
&= \left[f + \left(\frac{\partial \widehat{v}}{\partial x} - \frac{\partial \widehat{u}}{\partial y}\right)\right] V_s - W_s \frac{\partial \widehat{u}}{\partial z} - \frac{\partial J}{\partial x} + \widehat{F}_{d,x},
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \widehat{v}}{\partial t} &+ \widehat{u}\frac{\partial \widehat{v}}{\partial x} + \widehat{v}\frac{\partial \widehat{v}}{\partial y} + \widehat{w}\frac{\partial \widehat{v}}{\partial z} + f\widehat{u} + \frac{1}{\rho}\frac{\partial p^H}{\partial y} - \widehat{F}_{m,y} \\
&= -\left[f + \left(\frac{\partial \widehat{v}}{\partial x} - \frac{\partial \widehat{u}}{\partial y}\right)\right] U_s - W_s \frac{\partial \widehat{v}}{\partial z} - \frac{\partial J}{\partial y} + \widehat{F}_{d,y}.
\end{aligned}
$$

Graphic Interface of PALM coupler

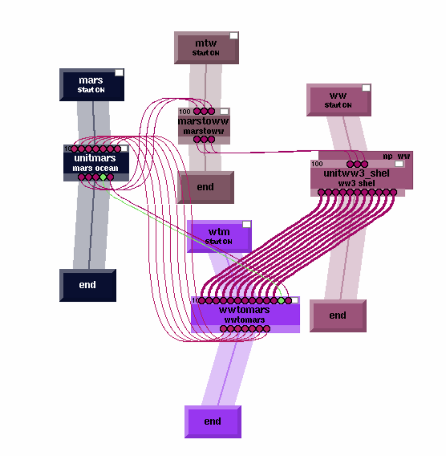


Figure: Prepalm map

Zonal velocity and sea surface elevation

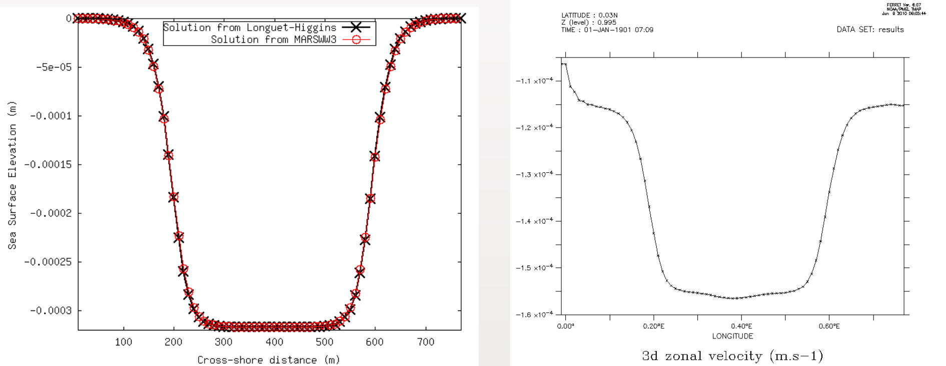


Figure: Comparison of sea surface elevation between the analytical solution from Longuet-Higgins and the numerical solution from the coupled model (left panel). Zonal velocity from the coupled model (right panel).

Without x-limits

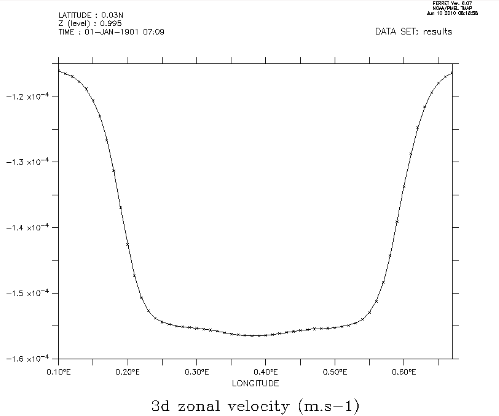


Figure: Zonal velocity from the coupled model without x-limits

Vertical structure

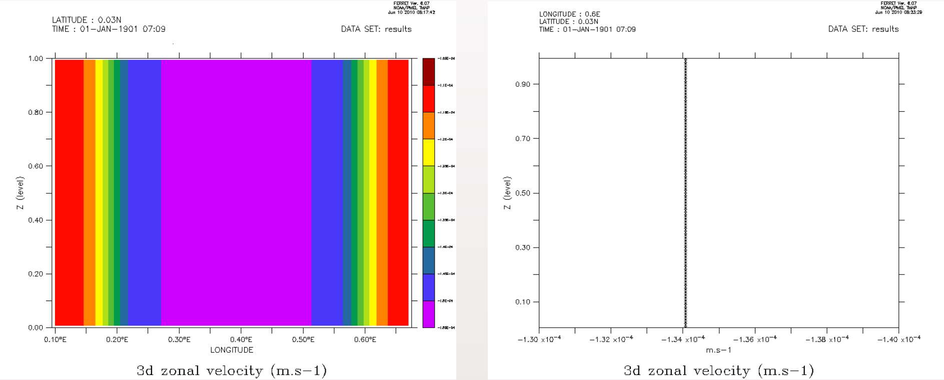


Figure: Zonal velocity without x-limits plotted over the water column (left panel). Vertical profile of zonal velocity (right panel).

Outline

In the future

- To test this new 3D version of coupled model in surf zone.
- Validation of this new 3D version of coupled model thanks to data of the ECORS 2008 experiment.
- To understand the undertow role in the exchange between the surf zone and the offshore zone: application to the Bay of Biscay.

The End.

Thank you.

Questions?