Bottom friction optimization for barotropic tidal modelling: twin experiments

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Introduction

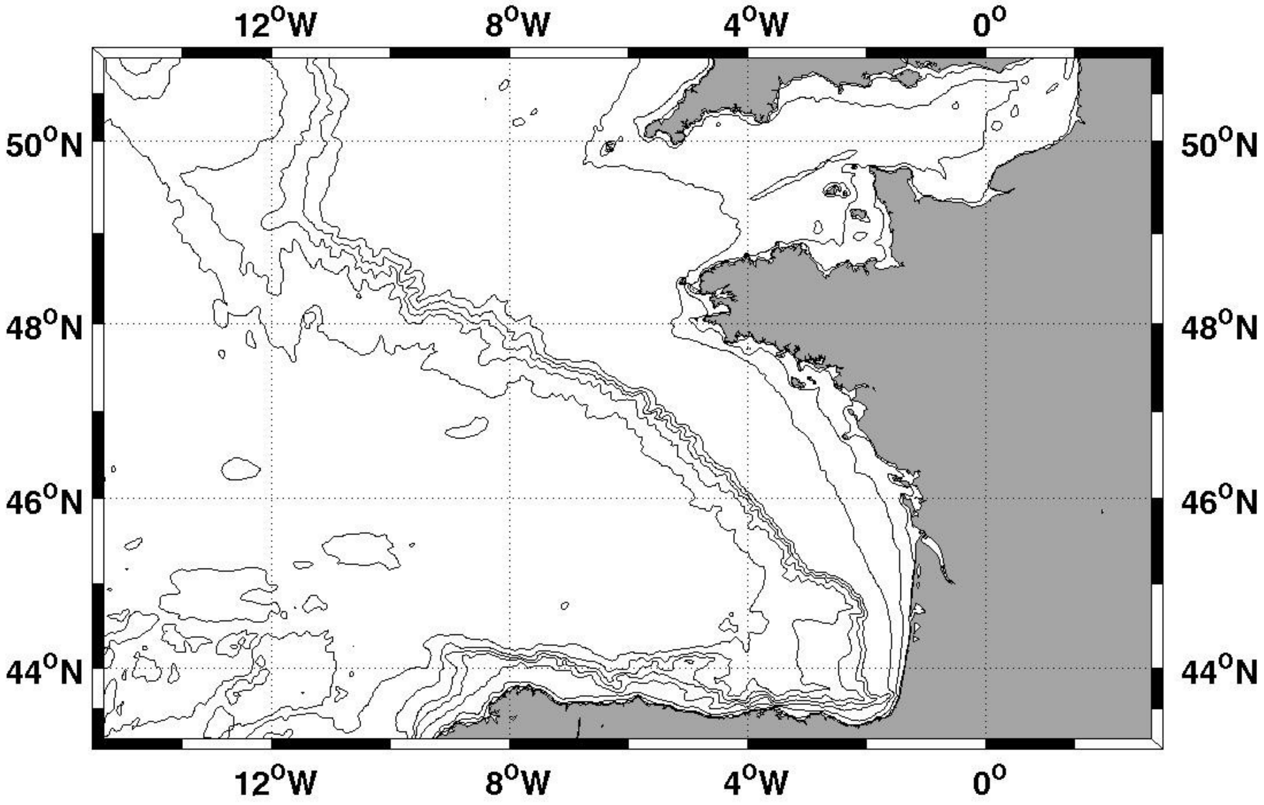
- **Bottom friction generally represented by a quadratic or a linear formulation :**

$$\vec{\tau} = -\rho C_D |\vec{u}| \vec{u}$$

$$\vec{\tau} = -\rho C_L \vec{u}$$

- **C_D and C_L often considered as constants**

- **Need for a more physically consistent approach in coastal areas**



Model Configuration

- **HYCOM in a pure barotropic configuration (one isopycnal layer)**

- **Horizontal resolution: 5.5 km**

- **Sea-surface elevations and 2D velocities (from the NEA2004 tidal atlas) prescribed along the open boundaries**

- **Bottom drag formulation :** $\vec{\tau} = \frac{-C_D |\vec{u}| \vec{u}}{H}$

Friction coefficient computation based on vertical integration of the turbulent velocity profile:

$$C_D = \left(\frac{\kappa}{\ln\left(\frac{H}{z_0}\right) - 1} \right)^2$$

κ : Von Kármán constant
z_0 : bottom roughness
H : water height

Algorithm for parameter estimation

- **Simultaneous Perturbation Stochastic Approximation (SPSA, *Spall* (1998))**
- **Iterative algorithm :** $\hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g}_k(\hat{\theta}_k)$
- **Gradient estimation with two loss function measurements :**

$$\hat{g}_{ki} = \frac{J(\hat{\theta}_k + c_k \Delta_{ki}) - J(\hat{\theta}_k - c_k \Delta_{ki})}{2 c_k \Delta_{ki}}$$

$$\Delta_{ki} = \begin{cases} 1, probability\ 1/2 \\ -1, probability\ 1/2 \end{cases}$$

→ Computational cost per iteration independent of the dimension problem

Results : twin experiments

- **“Observations” obtained with direct model integration**

- **Estimated parameter: z_0 (bottom roughness)**

- **Considerable problem dimension: parameter estimation only for some points (hereafter colocation points), and 2D reconstruction using interpolation**

- **Description of an iteration :**
  - 2 perturbed runs and associated loss function evaluations
  - Gradient computation and updating of the estimated parameter
  - 1 run with the new parameter distribution to evaluate the improvement or the deterioration in terms of loss function

Results : one degree of freedom

- **“Observations” obtained with a uniform z_0 distribution : $z_0=8mm$**

- **Only one tidal component: M2 (lunar semi-diurnal, most important component for the area)**

- **Uniform variation of z_0 (only one colocation point)**

- **Choices for a_k and c_k :**

$$c_k = \frac{0.005}{k^{1.2}+10} \qquad \hat{g}_{ki} = \frac{J(\hat{\theta}_k + c_k \Delta_{ki}) - J(\hat{\theta}_k - c_k \Delta_{ki})}{2 c_k \Delta_{ki}}$$

$$a_k = \frac{0.003}{k^{0.6}+10} \qquad \hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g}_k(\hat{\theta}_k)$$

Results : one degree of freedom

- **Assimilation window : 3.25 days**
  - Two days to enable the system to be in equilibrium with the imposed perturbation
  - Loss function computation during two periods of M2 (24.84 h)

- **Loss function calculated with SSH only (observations are supposed to be available at each grid point)**

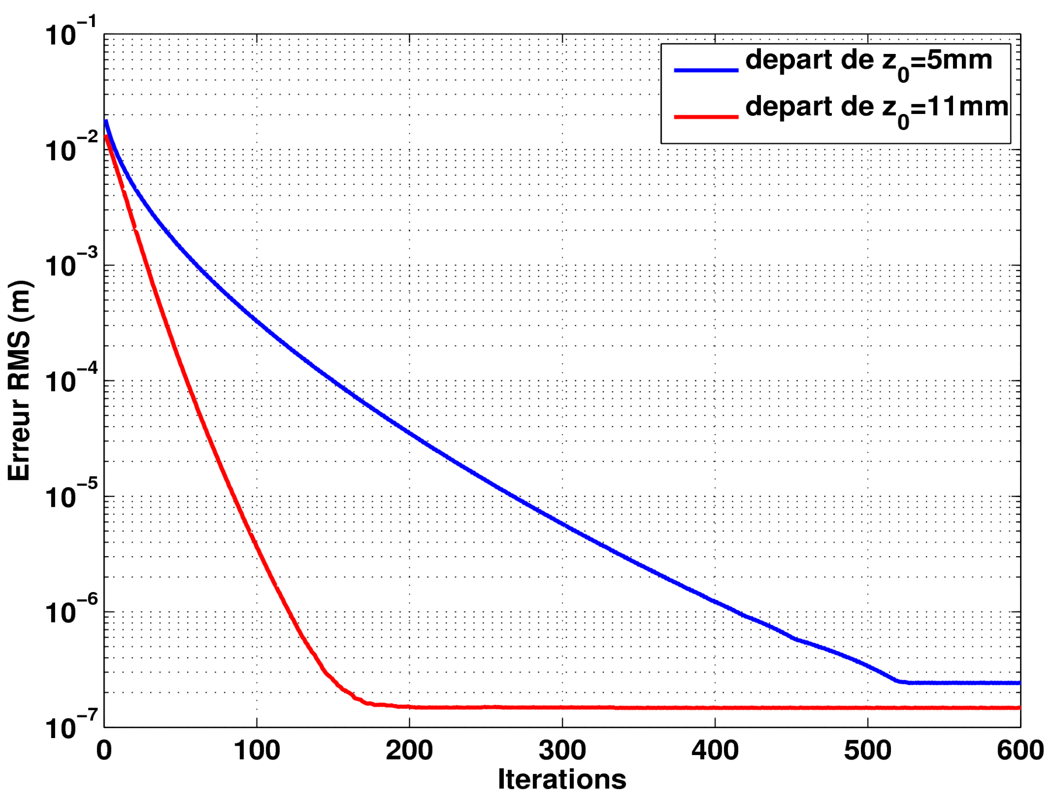
$$J = 0.5 \sum_{t=1}^{T} \sum_{i=1}^{N} \left(SSHmod_{i,t} - SSHobs_{i,t} \right)^2$$

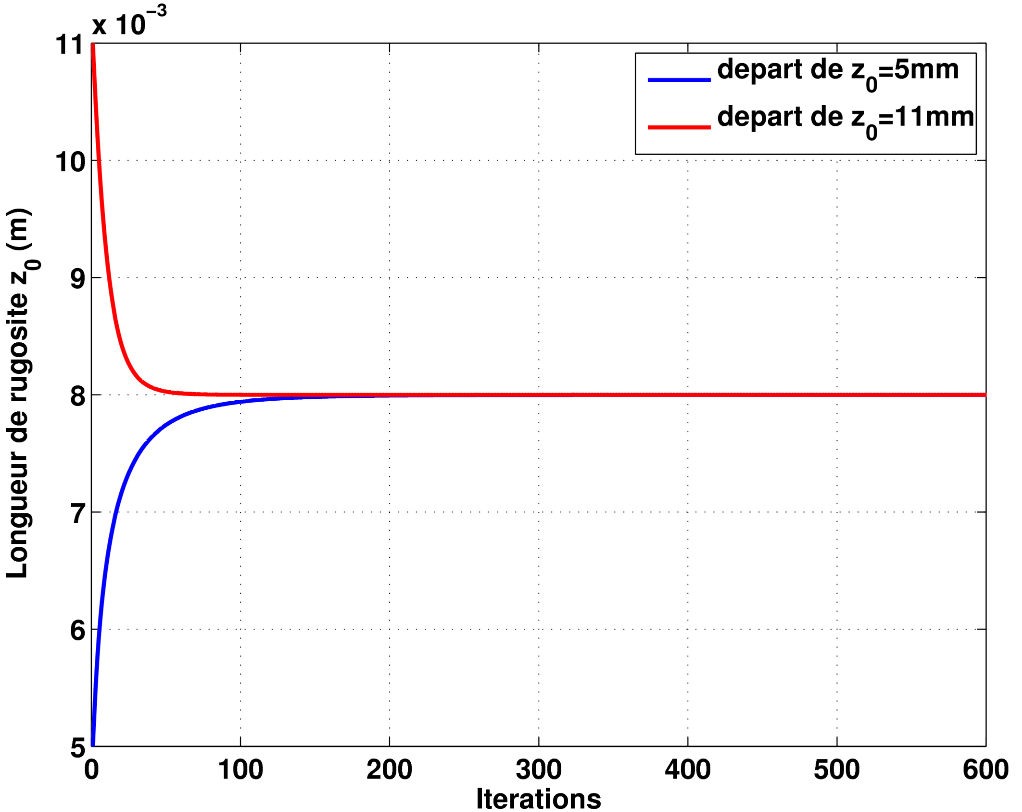
- **Need for a modification of the algorithm: gradient normalization when parameter is updated (gradient is normalized by the maximum gradient of the previous iterations)**

Results : one degree of freedom

Two cases:

- **start with z_0 = 5 mm**
- **start with z_0 = 11 mm**

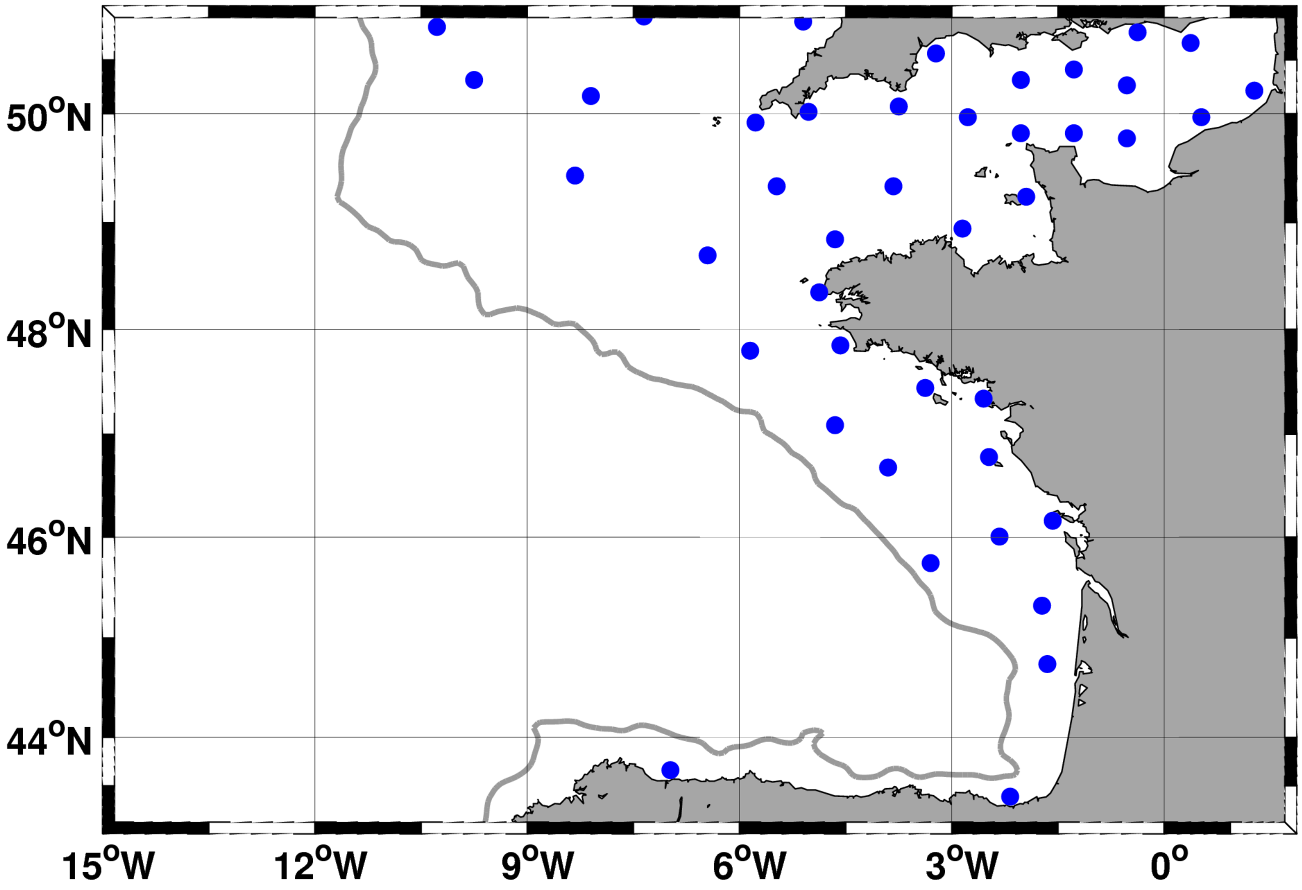




Final value : z_0 = 7.999966 mm

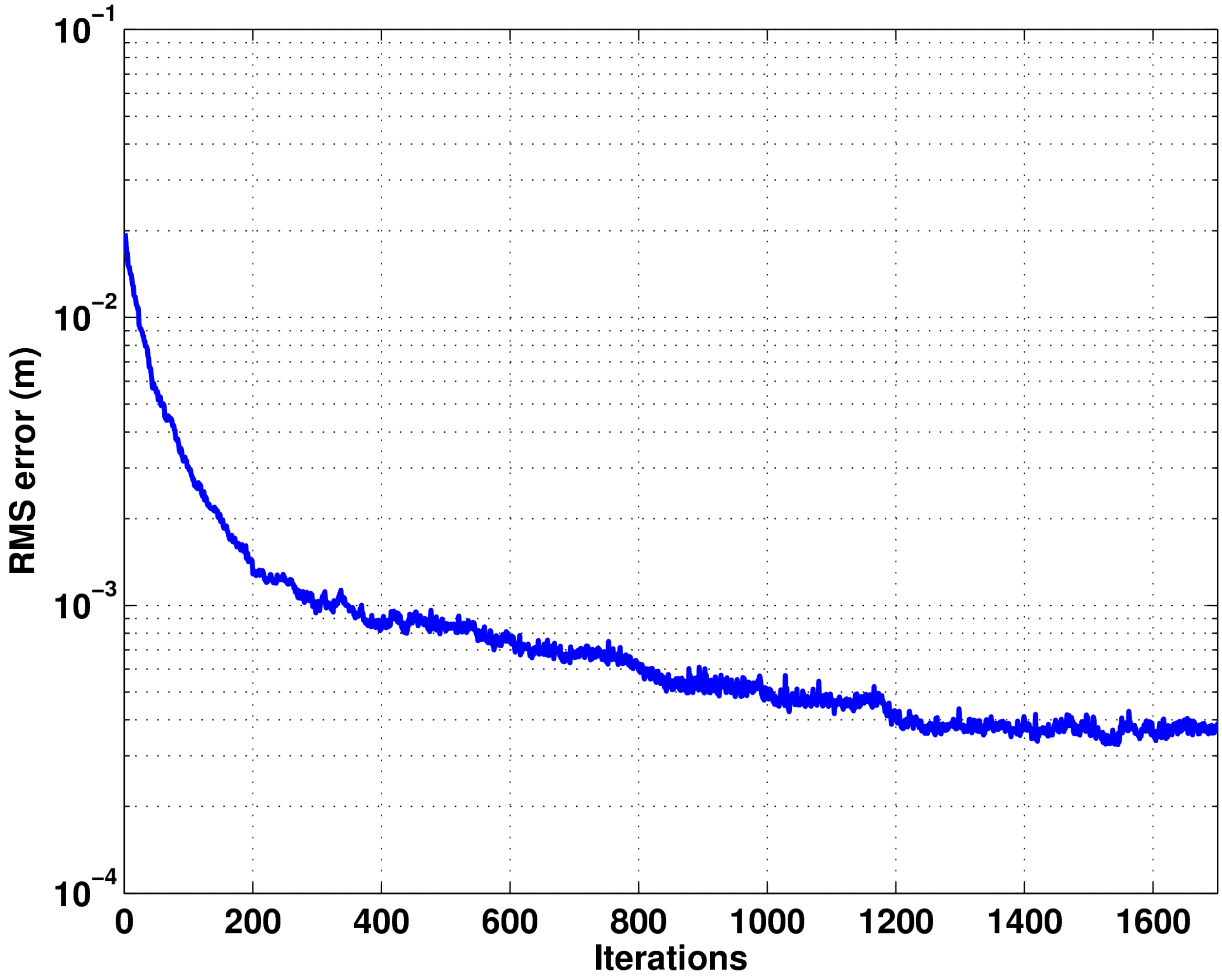
Final value : z_0 = 8.000005 mm

Results : several degrees of freedom

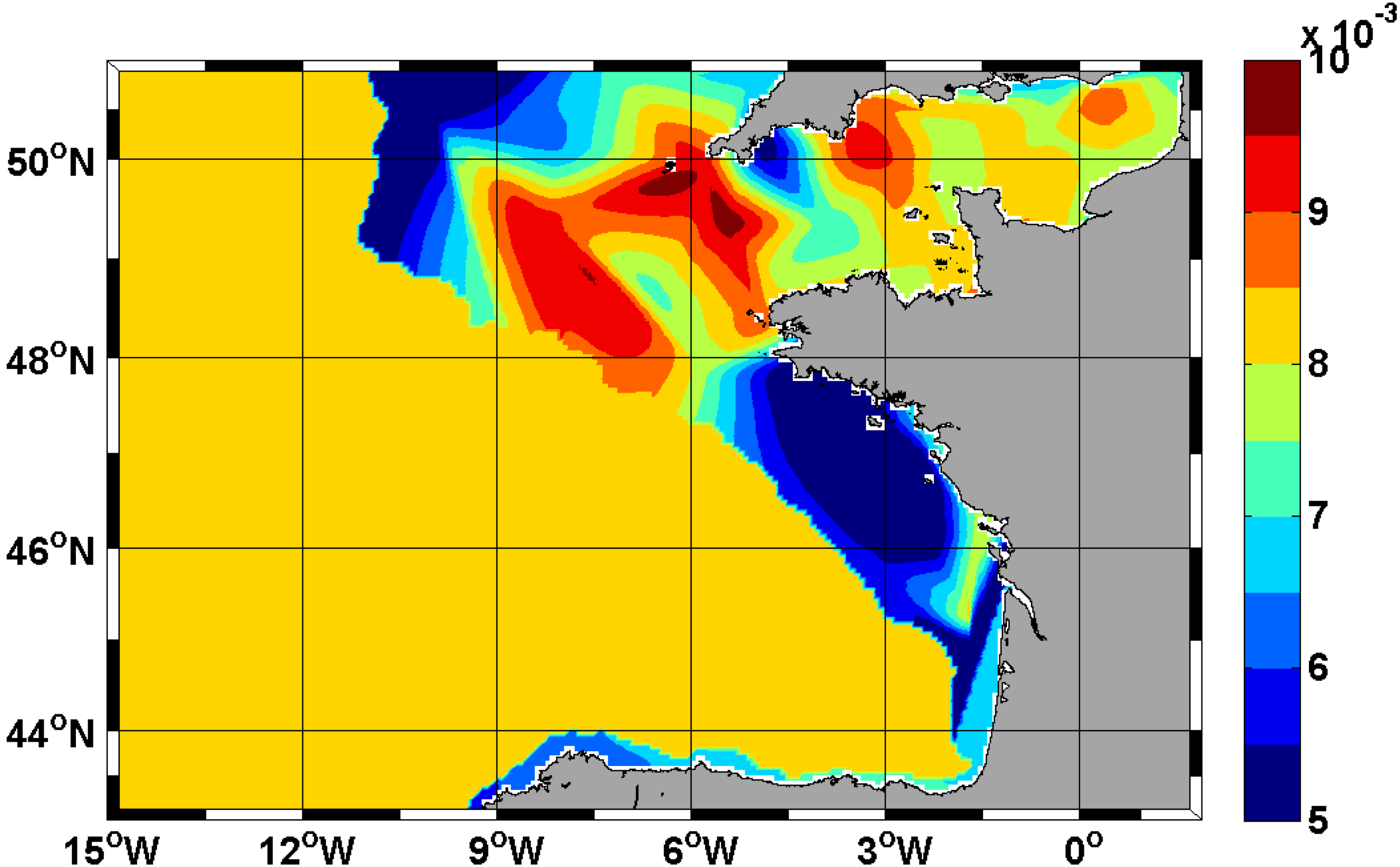


- 42 colocation points
- Start with uniform z_0 = 5mm
- Modified perturbations amplitudes (c_k): standard deviation of the parameter for the last N iterations (here N=20)

Results : several degrees of freedom

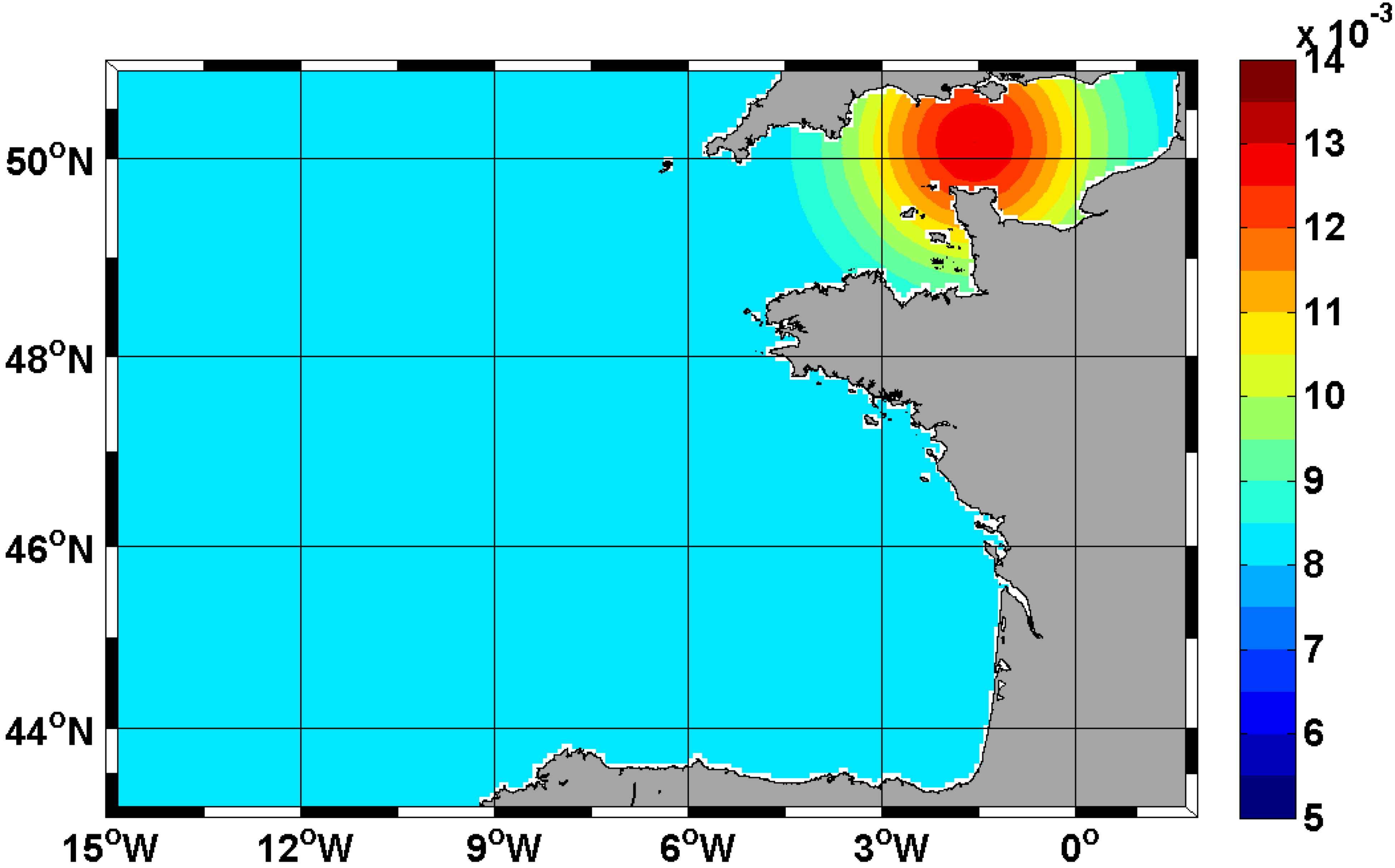


Results : several degrees of freedom

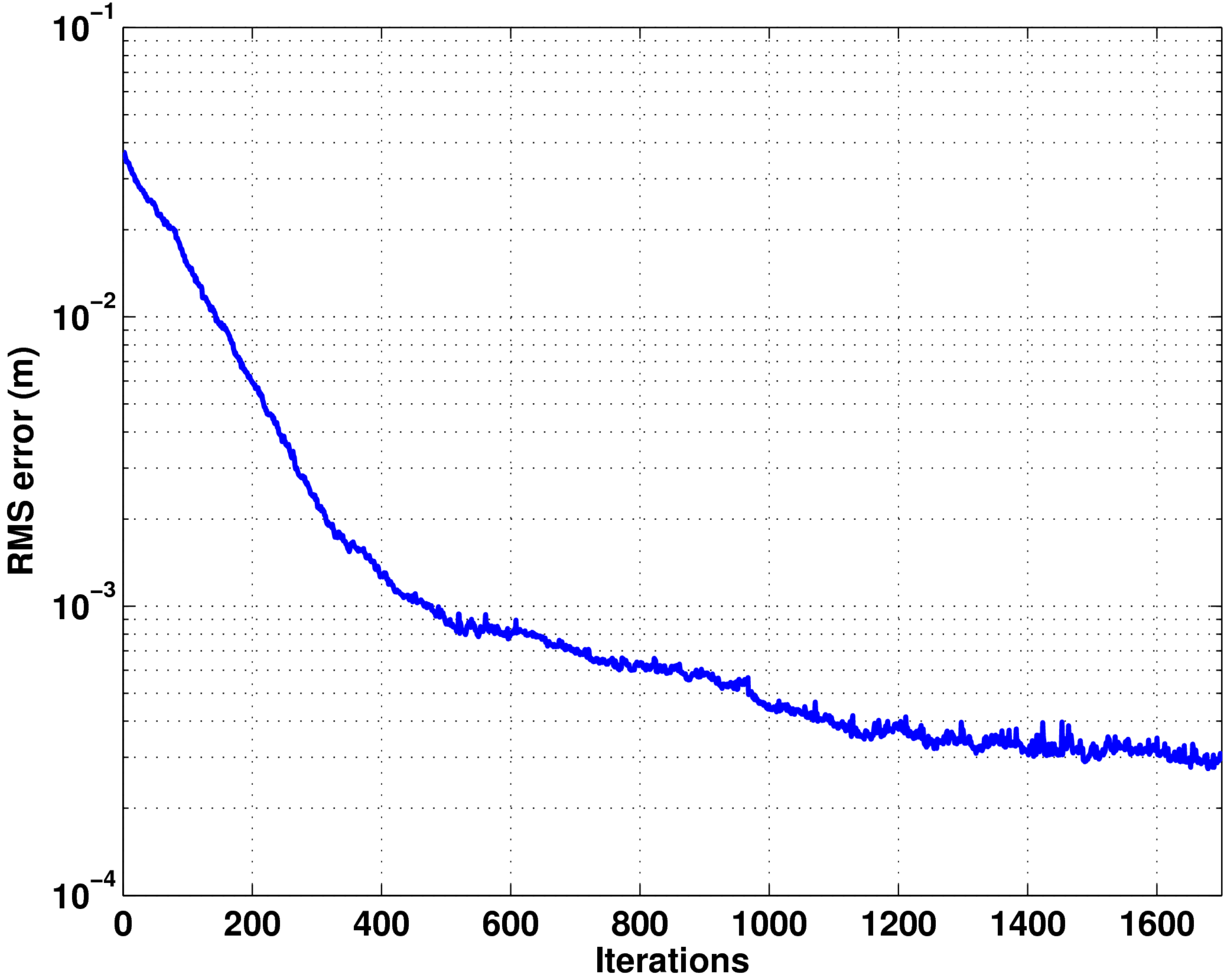


Results : several degrees of freedom

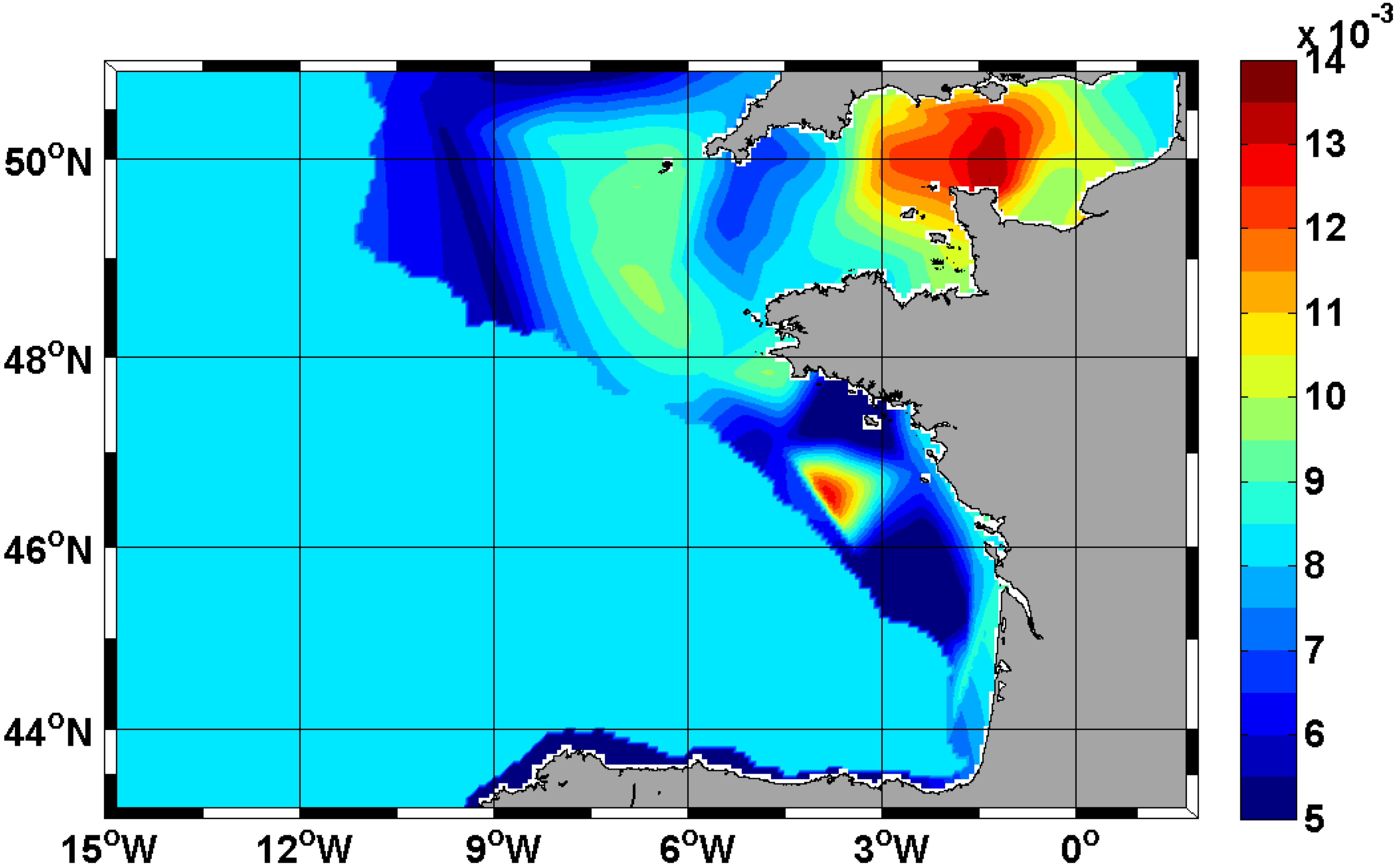
- "Observations" obtained with a variable z_0 distribution (gaussian distribution)



Results : several degrees of freedom



Results : several degrees of freedom



Conclusions and perspectives

- **SPSA algorithm easy to implement and appropriate for this study**

- **Realistic experiments:**
  - use of diverse data sources: tide gauges, ADCP moorings, satellite data (coastal products), HF radars
  - link between optimal z_0 distribution and a physical reality
  - quantification of the contribution of specific data, especially HF radars in the Iroise Sea